S.6 PURE MATHEMATICS ASSIGNMENT 1

Instructions to candidates

Attempt all questions in section A and section B

SECTION A

1. Prove that cos4A – cos4B – cos4C = 4sin2B sin2C cos2A – 1 given that A, B and Care angles of a triangle. (5 marks)
2. Given that y = *a*cosx + *b*sinx, where *a* and *b* are constants. Form a differential equation by eliminating the constants. (5 marks)
3. An error of 2% is made in measuring the circumference of a circle of radius r. what percentage error results in the area? (5 marks)
4. Use small changes to find the value of $\sqrt[3]{29}$. (5 marks)
5. Find the acute angle between the line $\vec{r} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} + t\begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix}$ and the plane

   $\mathbf{r}.\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} - 8 = 0$ (5 marks)
6. Given the vectors, **p** = 2**i** – **j** + 3**k**. Find,
   (i) The direction ratios,
   (ii) the direction cosines,
   (iii) the angle it makes with the direction **j.** (5 marks)
7. Determine the volume of the solid generated when an area enclosed by y = $\sec^2\theta$, the x-axis and $x = \frac{\pi}{4}$ is rotated through 2π about the x- axis. (5 marks)
8. Differentiate from first principles, cot*x*. (5 marks)

SECTION B

9. Triangle OAB has **OA** = **a** and **OB** = **b**. C is the point on OA such that **OC** = $\frac{2}{3}$ **a.** d is the mid-point of AB. When CD is produced it meets OB produced at E, such that **DE** = *n***CD** and **BE** = *k***b**. Express **DE** in terms of
   (i) *n*,**a** and **b** (6 marks)
   (ii) *k*,**a** and **b** (6 marks).
   Hence find the values of *n* and *k*

10. Evaluate

(a) $\int_0^{\pi/2} \sin 5x \cos 3x dx$ (6marks)

(b) $\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{9+4x^2} dx$ (6 marks)

11. (a) If y = $5x^4$ and is increased by 2% of its original value, find the corresponding percentage increase in y. (6 marks)

(b)Differentiate from first principles $x^{\frac{4}{3}}$ (6 marks)

12.(a) Find the value of λ if the line **r** = $\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix} + t\begin{pmatrix} 3\lambda \\ 5 \\ -1 \end{pmatrix}$ is parallel to the plane 3x – 4y +5z -1 = 0. (4 marks)

(b) Determine the vector equation of a plane through the points A(1, 0,2), B(-2, 1, 3) & C(4, 5,1). Hence write down the parametric and Cartesian equation of the plane. (8 marks)

13. Sketch the curve $y = \frac{(x+1)(x+2)}{(x+3)(x-1)}$ (12 marks)

14. (a) Solve the differential equation, $\frac{dy}{dx} - \frac{y}{x} = x$ given y(1) = 0. (5 marks)

(b) The rate of spread of coronavirus in a certain country is proportional to the number of people, p who have acquired the disease at time t. in December 2019, 200 people had got the disease, and two months later more 250 people got the disease. How many people will acquire the disease after three months? (7 marks)

15. (a) Given Z = 4 – 3i and $\overline{w} = 5i - 12$

Find; (i) $|ZW|$ (ii) Arg(zw), hence write zw in polar form. (5 marks)

(b) Determine the Cartesian equation of the locus, $Arg\left(\frac{z-1}{z+1}\right) = \frac{3\pi}{2}$ and represent it on an Argand diagram. (7 marks).

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